

## **MODULE-5- Physics of Animation**

### **Animation**

Animation is the process of displaying still images (drawings, models, or even puppets) in a rapid sequence to create the illusion of movement. Because our eyes can only retain an image for approx.  $\frac{1}{10}$  th of a second, when multiple images appear in fast succession, the brain blends them into a single moving image.

### **Frames and frames per second.**

A frame is a single image in a sequence of pictures. A frame contains the image to be displayed at a unique time in the animation. In general, one second of a video is comprised of 24 or 30 frames per second also known as FPS. The frame is a combination of the image and the time of the image when exposed to the view. An extract of frames in a row makes the animation.

### **The Taxonomy of Physics-Based Animation Methods**

At the highest level, the field of physics-based animation and simulation can roughly be subdivided into two large groups:

1. Kinematics is the study of motion without consideration of mass or forces.
2. Dynamics is the study of motion taking mass and forces into consideration.

kinematics and dynamics come in two flavors or subgroups:

1. Inverse is the study of motion knowing the starting and ending points.
2. Forward is the study of motion solely given the starting point.

### **Elucidate the Importance of Size & Scale, Weight and strength in animations (8M)**

#### **Size and Scale**

The size and scale of characters often play a central role in a story's plot.

We cannot imagine a Superman be without his height and bulging biceps? Some characters, like the Incredible Hulk, are even named after their body types.

We can equate large characters with weight and strength, and smaller characters with agility and speed. As it is noticeable in real life scenarios that, larger people and

animals do have a larger capacity for strength, while smaller critters can move and maneuver faster than their large counterparts.

When designing characters, we can run into different situations having to do with size and scale, such as:

1. Human or animal-based characters that are much larger than we see in our everyday experience. Superheroes, Greek gods, monsters,
2. Human or animal-based characters that are much smaller than we are accustomed to, such as fairies and elves.
3. Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.
4. Characters that are child versions of older characters. An example would be an animation featuring a mother cat and her kittens. If the kittens are created and animated with the same proportions and timing as the mother cat, they won't look like kittens; they'll just look like very small adult cats.

### Proportion and Scale

Creating a larger or smaller character is not just a matter of scaling everything about the character uniformly.

Example: When we scale a cube, its volume changes much more dramatically than its surface area. Let us say each edge of the cube is 1 unit length. The area of one side of the cube is 1 square unit, and the volume of the cube is 1 cubed unit.

If we double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times, and its volume increases by 8 times. While the area increases by squares as we scale the object, the volume changes by cubes.

### Wight and strength

Body weight is proportional to volume. The abilities of our muscles and bones, however increase by area because their abilities depend more on cross-sectional area than volume.

To increase a muscle or bone's strength, we need to increase its cross-sectional area.

To double a muscle's strength, for example, you would multiply its width by  $\sqrt{2}$ .

To triple the strength, multiply the width by  $\sqrt{3}$ .

Since strength increases by squares and weight increases by cubes, the proportion of a character's weight that it can lift does not scale proportionally to its size.

Let us take an example of a somewhat average human man. At 6 feet tall, he weighs 180 pounds and can lift 90 pounds. He can lift half his body weight.

If we scale up the body size by a factor of 2, the weight increases by a factor of 8. Such a character could then lift more weight. But since he weighs more than 8 times more than he did before, he cannot lift his arms and legs as easily as a normal man. Such a giant gains strength, but loses agility.

**Discuss the timing in Linear motion, Uniform Motion, ease in (Slow in ) and ease out (slow out) (8M)**

Timing animation refers to the duration of an action.

In animation, timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion.

**Line of action:** Individual drawings or poses have a **line of action**, which indicates the visual flow of action at that single image.



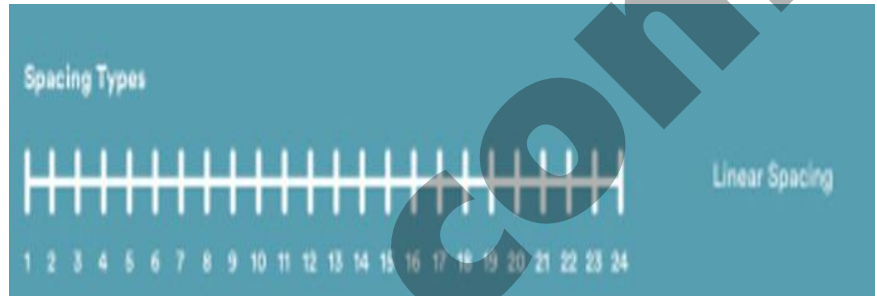
**Path of action** Motion has a **path of action**, which indicates the path along which the object or character moves. The path of action refers to the object's motion in space.



An object moving with linear motion might speed up, slow down or move with a constant speed and it follows a linear path.

- 1) **Uniform motion:** It is the easiest to animate because the distance the object travels between frames is always the same.

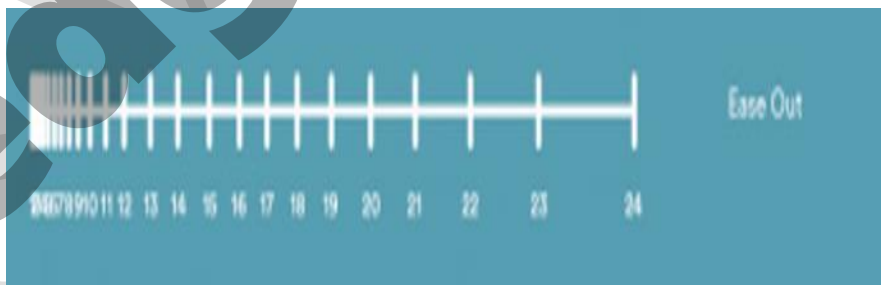
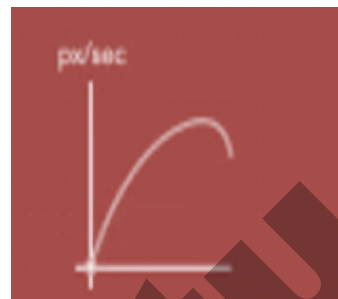
The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.



## 2) Ease out / Speed up

The object is speeding up i.e its speed increases gradually, often from a still position.

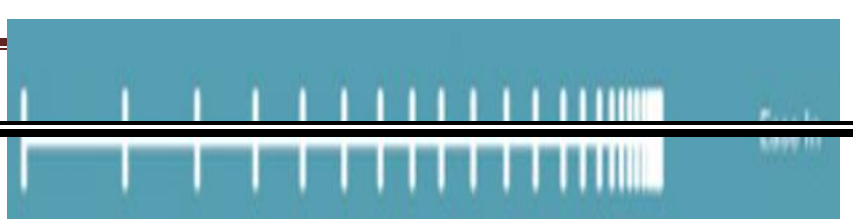
The frames are located such that, initially the frames are closely spaced with gradual increase in the spacings.



## 3) Ease in/ Slowed down.

The object is slowing down, its speed decreases gradually often in preparation for stopping.

The frames are located such that, initially the frames are widely spaced with gradual decrease in the spacings of the frames.

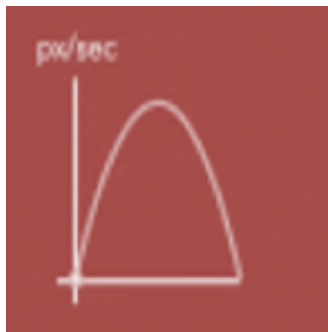


#### **4) Ease out- Ease in or Ease-Ease.**

It is the combination of speed up and slowed down. That is the object initially gets speed up initially and finally comes to still position with slowing down.

In the beginning the frames are located such that, initially the frames are closely spaced with gradual increase in the spacings up to middle position.

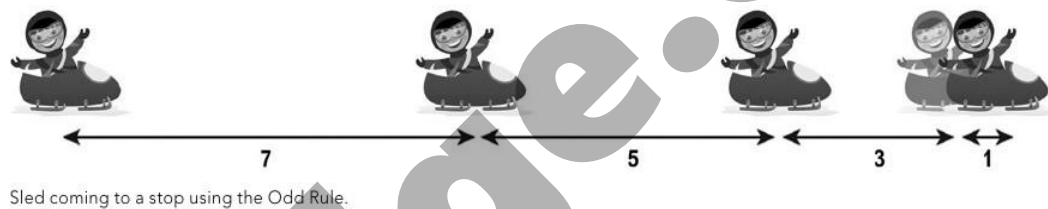
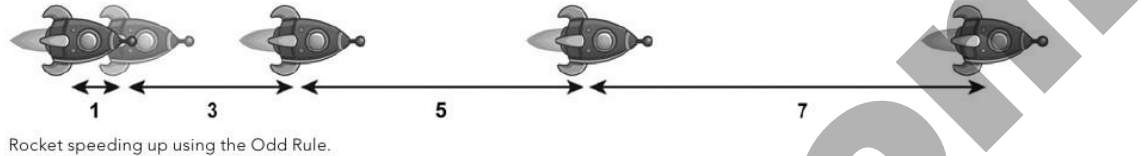
From the middle position onwards, the frames are widely spaced with gradual decrease in the spacings of the frames towards the still position.



#### **Illustrate the odd rule and odd rule multipliers with a suitable example (8M)**

- When acceleration is constant, The Odd Rule is used (Simple Pattern of Odd Numbers) to time the frames.
- Between consecutive frames, the distance moved by the object is a multiple of an odd number.
- For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.
- For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.

- The Odd Rule is a multiplying system based on the smallest distance (base distance) travelled between two frames in the sequence
- Base distance** :For a slow-out is the distance between the first two frames and for a slow-in: the distance between the last two frames is called as the **base distance**.

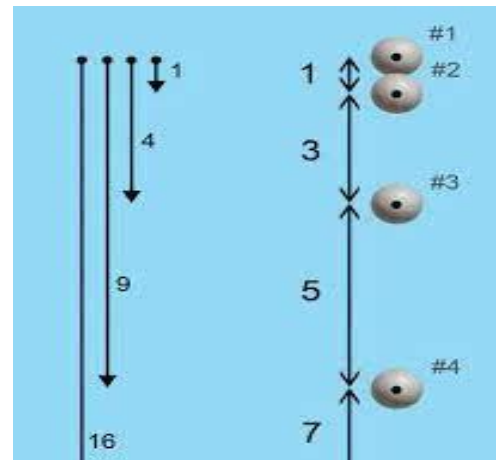


Odd Rule Multipliers can be used to calculate the distance from the first frame to the current frame and use these distances to place the object on specific frames

Odd multipliers for Consecutive Frames =  $((\text{Frame\#} - 1) \times 2 - 1)$

Multiplier for distance from first frame to current frame =  $(\text{Current Frame\#} - 1)^2$

Multiply by base distance to get distance between:		
Frame #	Consecutive frames	First frame and this frame
1	n/a	0
2	1	1
3	3	4
4	5	9
5	7	16
6	9	25
7	11	36



**Odd rule scenarios**

- Base Distance Known Speeding up : Base Distance \* odd rule multipliers from the first frame
- Base Distance Known Slowing Down : The base distance \* Odd rule multipliers backwards.
- Total Distance and Number of Frames Known, Speeding Up :

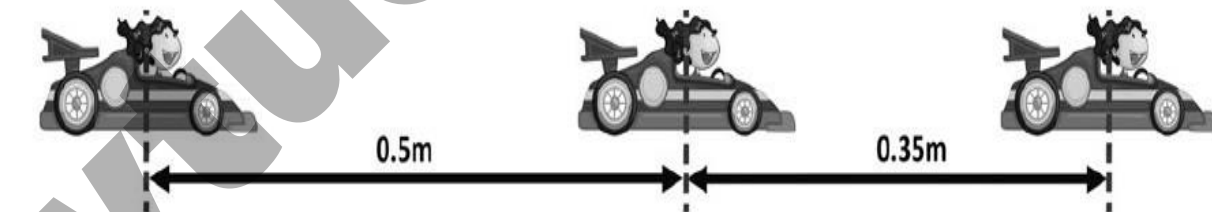
$$\text{Base distance} = \frac{\text{Total distance}}{(\text{Last frame number} - 1)^2}$$

**Example:** Suppose there is a jump push (take-off) with constant acceleration over 5 frames, and the total distance travelled is 0.4m. Using the formula above, we find the base distance.

$$\text{Base distance} = \frac{0.4\text{m}}{(5 - 1)^2} = \frac{0.4\text{m}}{16} = 0.025\text{m}$$

**A slowing in object in an animation has a first frame distance of 0.5 m and the slow in frame 0.35m. Calculate the base distance and the number of frames in sequence (5M)**

For the given example the illustration can be written as

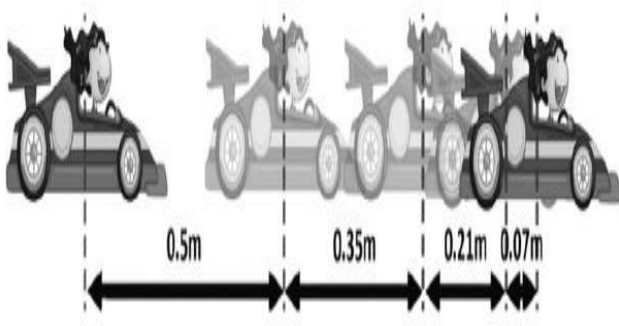


One of the features of the Odd Rule is that the base distance is always half the difference between any two adjacent distances.

$$\text{Consecutive Frame Multiplier} = \frac{\text{First Distance}}{\text{Base Distance}} = \frac{0.5}{0.07} = 7$$

Thus, Consecutive Frame Multiplier '7' Corresponds to '4' Frames





Frame #	Consecutive frame multiplier	Distance from previous frame
1	7	$7 * 0.07\text{m} = 0.5\text{m}$
2	5	$5 * 0.07\text{m} = 0.35\text{m}$
3	3	$3 * 0.07\text{m} = 0.21\text{m}$
4	1	$1 * 0.07\text{m} = 0.07\text{m}$

## Describe Jumping, parts of Jump and Jump Magnification (8M)

**A jump** is an action where the character's entire body is in the air, and both the character's feet leave the ground at roughly the same time.

*A jump action includes a take-off, free movement through the air, and a landing.*

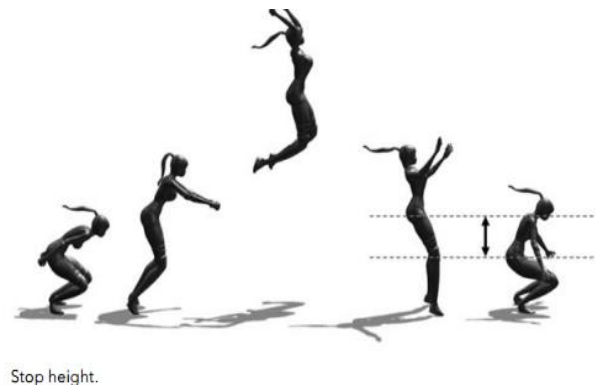
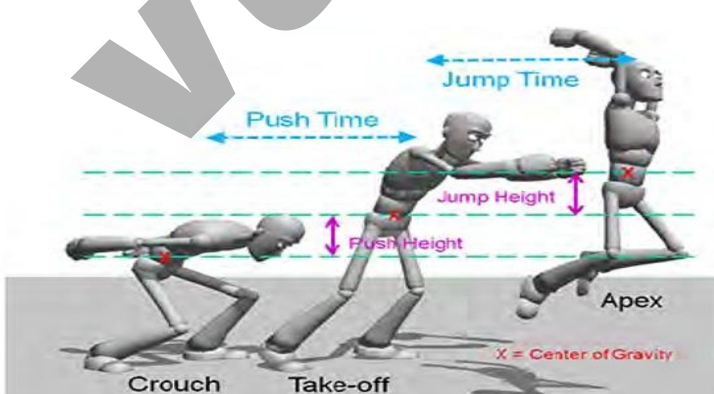
### Parts of Jump :

**Crouch**—A squatting pose taken as preparation for jumping.

**Take off**—Character pushes up fast and straightens legs with feet still on the ground. The amount of time (or number of frames) needed for the push is called the push time.

**In the air**— Both the character's feet are off the ground, and the character's CG moves in a parabolic arc as any free-falling body would.

**Landing**—Character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.





**Push height:** The distance between Center of gravity (CG) in crouch position to CG of Take off position

**Jump Height:** The distance between CG in takeoff position to CG of position at air.

**Stop Height:** The distance between CG in Landing position to CG of Crouch position during landing.

$$\text{Jump Height} = 1.2\text{m}$$

$$\text{Jump Time} = t = \sqrt{\frac{2h}{g}} = 0.5\text{s}$$

$$\text{Jump Time at 30 fps} = 0.5 \times 30 = 15 \text{ frames.}$$

## **Jump Magnification**

Jump Magnification is in fact an exact ratio that tells one how much the character has to accelerate against gravity to get in to the air.

**Push time:** The number of frames required to move from 'crouch position' to 'Take off position'.

**Jump time:** The number of frames required to move from 'Take off position' to 'In air position'.

**Stop time:** The number of frames required to move from 'In air position' to 'Landing position'.

$$JM = \frac{\text{Jump Time}}{\text{Push Time}} = \frac{\text{Jump Height}}{\text{Push Height}} = \frac{\text{Push Acceleration}}{\text{Jump Acceleration}} = \frac{\text{Push Acceleration}}{\text{Gravitational Acceleration}}$$

### **Example:**

Push Time: 5 frames

Push Height: 0.4m

Stop Height: 0.5m

Stop Time =  $(5 * 0.5) / 0.4 = 6$  frames

## Define Strides and Gait.

### Walking

Walks feature all the basics of mechanics while including personality. The ability to animate walk cycles is one of the

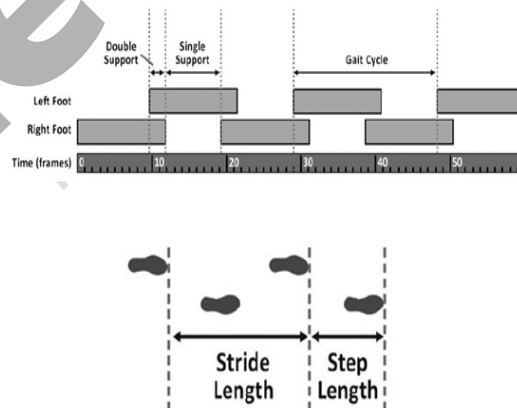
most important skills a character animator needs to master.

### Strides and Steps

A step is one step with one foot. A stride is two steps, one with each foot. Stride length is the distance the character travels in a stride, measured from the same part of the foot. Step and stride length indicate lengthwise spacing for the feet during a walk.

**Gait** is the timing of the motion for each foot, including how long each foot is on the ground or in the air.

During a walk, the number of feet the character has on the ground changes from one foot (single support) to two feet (double support) and then back to one foot. You can plot the time each foot is on the ground to see the single and double support times over time. A normal walking gait ranges from  $1/3$  to  $2/3$  of a second per step, with  $1/2$  second being average.



## Statistical Physics for Computing

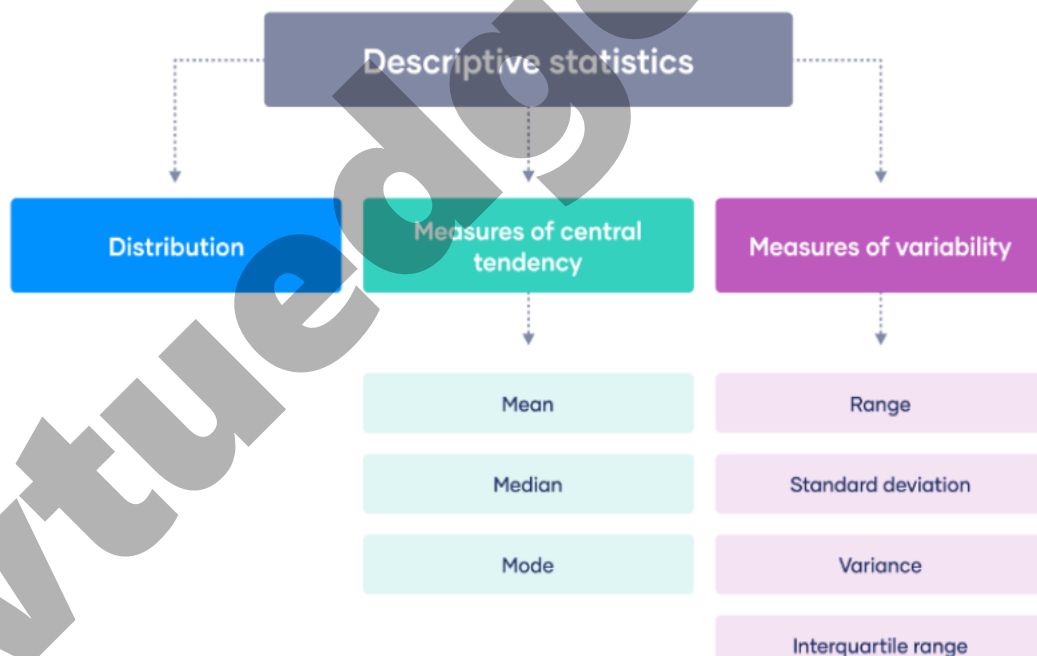
## Distinguish between Descriptive Statistics and Inferential statistics

**Statistical physics** is a branch of physics that evolved from a foundation of statistical mechanics, which uses methods of probability theory and statistics, particularly the mathematical tools for dealing with large populations and approximations, in solving physical problems.

**Descriptive statistics:** The term “descriptive statistics” refers to summarizing and organizing the characteristics of a data set. A data set is a collection of responses or observations from a sample or entire population.

In quantitative research, after collecting data, the first step of statistical analysis is to describe characteristics of the responses, such as the average of one variable (e.g., age), or the relation between two variables (e.g., age and creativity).

Descriptive statistics comprises three main categories – Frequency Distribution, Measures of Central Tendency, and Measures of Variability.



## Inferential Statistics:

Inferential Statistics is a method that allows us to use information collected from a sample to make decisions, predictions, or inferences from a population. The major inferential statistics are based on statistical models such as Analysis of Variance, chi-square test, student's t distribution, regression analysis, etc.

### Methods of inferential statistics:

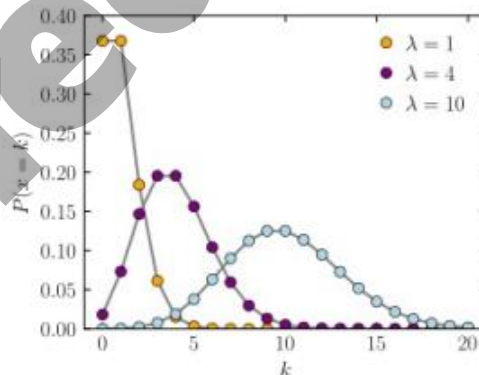
- Estimation of parameters
- Testing of hypothesis

### Explain the Poisson's distribution with an example

**Poisson Distribution** If the probability  $p$  is so small that the function has significant value only for very small  $k$ , then the distribution of events can be approximated by the Poisson Distribution. Probability mass function A discrete Random variable  $X$  is said to have a Poisson distribution, with parameter  $\lambda$ , if it has a probability Mass Function given by

$$f(k; \lambda) = P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Here  $k$  is the number of occurrences,  $e$  is Euler's Number,  $!$  is the factorial function. The positive real number  $\lambda$  is equal to the expected value of  $X$  and also to its Variance. The Poisson distribution may be used in the design of experiments such as scattering experiments where a small number of events are seen.



**Example of probability for Poisson distributions** On a particular river, overflow floods occur once every 100 years on average. Calculate the probability of  $k = 0, 1, 2, 3, 4, 5$ , or 6 overflow floods in a 100 year interval, assuming the Poisson model is appropriate. Because the average event rate is one overflow flood per 100 years,  $\lambda = 1$

$$f(K, \lambda) = P(X = K) = \frac{\lambda^K e^{-\lambda}}{K!}$$

$$P(K \text{ overflow floods in 100 years}) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^K e^{-1}}{K!}$$

$$P(K=0 \text{ overflow floods in 100 years}) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} = 0.368$$

$$P(K=1 \text{ overflow floods in 100 years}) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^1 e^{-1}}{1!} = \frac{e^{-1}}{1} = 0.368$$

$$P(K=2 \text{ overflow floods in 100 years}) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} = 0.184$$

## Discuss the modelling probability for proton decay

### Proton decay

Proton decay is a rare type of radioactive decay of nuclei containing excess protons, in which a proton is simply ejected from the nucleus. The mechanism of the decay process is very similar to alpha decay. Proton decay is also a quantum tunneling process.

#### Modeling the Probability for Proton Decay

The probability of observing a proton decay can be estimated from the nature of particle decay and the application of Poisson Statistics. The number of protons  $N$  can be modeled by the decay eq

$$N = N_0 e^{-\lambda t}$$

Where:

$N_0$ : is the initial quantity of the element

$\lambda$ : is the radioactive decay constant

$t$ : is time

$N(t)$ : is the quantity of the element remaining after time  $t$ .

Here  $\lambda = 1/t = 10^{-33} / \text{year}$  is the probability that any given proton will decay in a year.

Since the decay constant  $\lambda$  is so small, the exponential can be represented by the first two terms of the Exponential Series.

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$e^{-\lambda t} = 1 - \lambda t$$

Most recently the experiment on proton decay has been done by Super Kamiokande, Japan which started observation in 1996. It is a large water Cherenkov detector which is the most sensitive detector in the world used to examine proton decay with the huge source with  $7.5 \times 10^{33}$  protons

For one year of observation, the number of expected proton decays is then

$$N_0 - N = N_0 \lambda t$$

$$= (7.5 \times 10^{33} \text{ protons})(10^{-33} / \text{year})(1 \text{ year})$$

$$N_0 - N = 7.5$$

$$N_0 - N = N_0(1 - \lambda t)$$

$$= (7.5 \times 10^{33} \text{ protons})(10^{-33} / \text{year})(1 \text{ year})$$

$$N_0 - N = 7.5$$

Proton decay has not been detected experimentally till now probably because of fact that the event is extremely rare. Assuming that  $\lambda = 3$  observed decays per year is mean, then the Poisson distribution function tells us that the probability for zero observations of decay is

$$P(K) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{3^0 e^{-3}}{0!} = 0.05$$

This low probability for a null result suggests that the proposed lifetime of  $10^{33}$  years is too short.

## **Discuss the salient features of normal distribution using bell curves**

### **Normal Distribution:**

The bell curve is a normal probability distribution of variables plotted on the graph and is like a bell shape where the highest or top point of the curve represents the most probable event out of all the series data.

#### **CHARACTERISTICS**

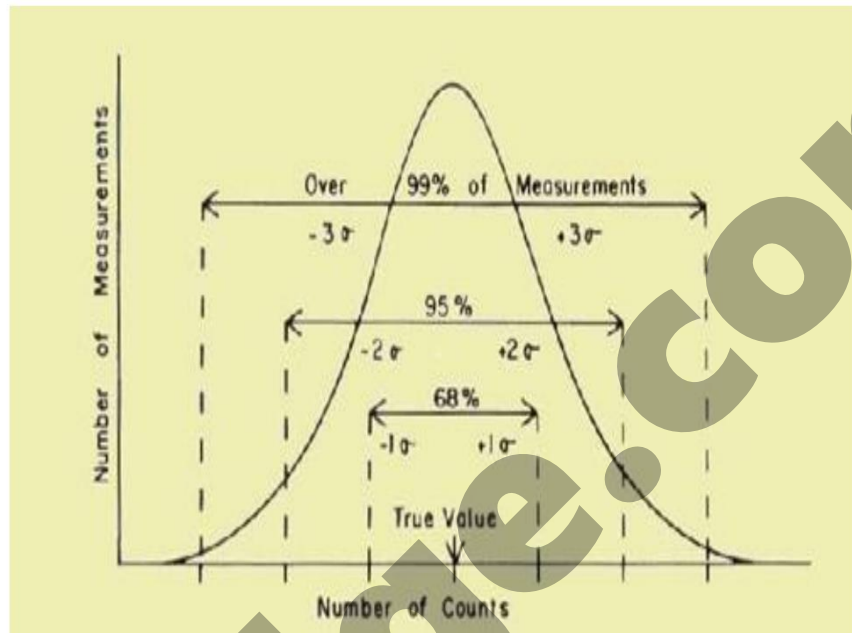
1. The Normal Curve is Symmetrical: The normal probability curve is symmetrical around its vertical axis called ordinate which represents the mean of distribution. The symmetry about the ordinate at the central point of the curve implies that the size, shape, and slope of the curve on one side of the curve is identical to that of the other. In other words, the left and right halves of the middle central point are mirror images, as shown in the figure given here.
2. The Normal Curve is Unimodal: Since there is only one maximum point in the curve, thus the normal probability curve is unimodal, i.e. it has only one mode.
3. The Normal Curve is Bilateral: The total area under the curve is 1, the 50% area of the curve lies to the left side of the maximum central ordinate and 50% of the area lies to the right side. Hence the curve is bilateral.
4. The Normal Curve is a mathematical model in behavioral Sciences: This curve is used as a measurement scale. The measurement on it of this scale is  $\pm 1\sigma$  (the unit standard deviation).

**Standard Deviations:** The standard normal distribution is a normal probability distribution that has a mean of 0 and a standard deviation of 1. The Standard Deviation

is a measure of how spread-out numbers are. As per 3 sigma rule of normal distribution,

- I. 68% of values are within 1 standard deviation of the mean.
- II. 95% of values are within 2 standard deviations of the mean.
- III. 99.7% of values are within 3 standard deviations of the mean





**Mention the general pattern of Monte-Carlo Method and hence determine the value of pi.**

### **Monte-Carlo Method:**

Monte Carlo Simulation, also known as the Monte Carlo Method or a multiple probability simulation, is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. The Monte Carlo Method was invented by John von Neumann and Stanislaw Ulam during World War II to improve decision-making under uncertain conditions. It was named after a well-known casino town, called Monaco.

The statistical method of understanding complex physical or mathematical systems by using randomly generated numbers as input into those systems to generate a range of solutions.

How to use Monte Carlo methods

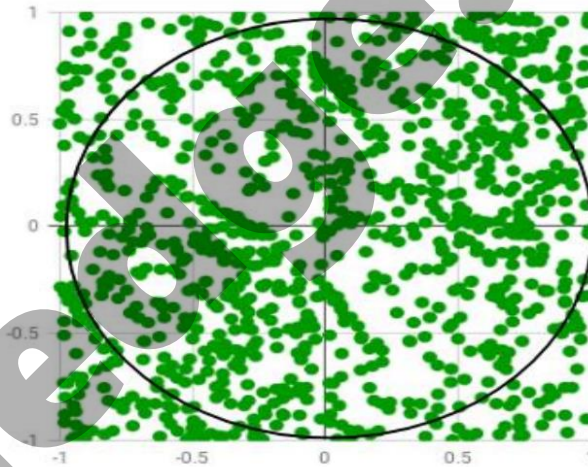
1. Define a domain of possible inputs

2. Generate inputs randomly from a probability distribution over the domain
3. Perform a deterministic computation on the inputs
4. Aggregate the results

### Estimation of Pi

- The idea is to simulate random (x, y) points in a 2-D plane with the domain as a square of side  $2r$  units centered on (0,0).
- Imagine a circle inside the same domain with the same radius  $r$  and inscribed into the square.
- We then calculate the ratio of the number of points that lay inside the circle and the total number of generated points.

Refer to the image below:



We know that the area of the circle  $\pi r^2$ , while that of square  $4r^2$ . The ratio of these two areas is as follows:

$$\frac{\text{area of the circle}}{\text{area of the square}} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Now for a very large number of generated points  $\frac{\text{no of points generated inside the circle}}{\text{no of points generated inside the square}} = \frac{\pi}{4}$

$$4 * \frac{\text{no of points generated inside the circle}}{\text{no of points generated inside the square}} = \pi$$